Inelastic Scattering of 18.2-MeV Protons by Deuterons*

ROBERT E. WARNER†
Antioch College, Yellow Springs, Ohio
(Received 30 July 1963)

Measurements of the inelastic cross section $d^3\sigma/d\Omega_1 d\Omega_2 dE_1$ for breakup of deuterons by 18.2-MeV protons were made by detecting both breakup protons in coincidence. *p-d* breakup events were identified from two-dimensional pulse-height spectra. Typical cross sections ranged from 3 mb/sr²–MeV at $\theta_1=-\theta_2=15°$ to 0.6 mb/sr²–MeV at $\theta_1=-\theta_2=60°$. The data are in substantial disagreement with the predictions of two theories based upon the impulse approximation: the zero-range theory of Frank and Gammel, and the spectator model of Kuckes, Wilson, and Cooper. Reasons for the disagreement are discussed.

I. INTRODUCTION

In this article we report coincidence measurements of the inelastic differential cross section $d^3\sigma/d\Omega_1 d\Omega_2 dE_1$ for breakup of deuterons by 18.2 MeV protons. Here, $\Delta\Omega_1$ and $\Delta\Omega_2$ are the solid angles in which the two breakup protons from the reaction are detected, and E_1 is the laboratory kinetic energy of one proton. Related measurements have been reported at nucleon bombarding energies above 140 MeV,[1–3] and below 15 MeV a number of measurements of the inelastic cross section $d^2\sigma/d\Omega dE$ for the detection of a single particle have been made.[4–7] Theoretical treatments of inelastic nucleon-deuteron scattering have been given by Frank and Gammel[8] and by Kuckes, Wilson, and Cooper[1]; the present data are compared with the predictions of both of these theories.

The nucleon-deuteron breakup reactions are of interest since they are the only nuclear reactions in which only three nucleons are involved. Since an exact theoretical treatment of the three-nucleon problem is not available, the questions of primary theoretical interest concern the approximations which may be made and the mechanism through which breakup occurs. Most theoretical treatments, including the two discussed here, have employed the impulse approximation.[9] The Frank-Gammel theory also assumes zero-range nucleon-nucleon interaction potentials; it then follows that only the even parity *n-p* potentials contribute to the scattering. This theory successfully predicts the proton yields from the *p-d* reaction at 9.66 MeV and the *n-d* reaction at 14.1 MeV,[8] and gives the correct integrated yield (but not the correct energy distribution) of neutrons produced at zero degrees in the *p-d* reaction at 7 MeV.[4] Since the contributions from the *n-n* and *p-p* potentials vanish in this treatment, it fails to predict the high energy peaks which are observed in the neutron spectrum from the *p-d* reaction at 9 MeV[5] and the proton spectrum from the *n-d* reaction at 14 MeV;[6] these peaks result from the final state interaction between identical nucleons.[10]

At higher energies, the spectator model of Kuckes, Wilson, and Cooper[1] often gives at least qualitatively correct predictions for nucleon-deuteron inelastic scattering. Again the impulse approximation is made and it is also assumed that the incident nucleon interacts with only one of the nucleons in the deuteron, i.e., the nuclear force exerted by the incident nucleon upon the other nucleon (called the spectator particle) is ignored. Thus the collision differs from an elastic nucleon-nucleon collision only in that the target nucleon is not at rest but has the same momentum distribution as a nucleon in the deuteron. At 145 MeV, the measured cross sections for quasielastic *p-p* scattering from deuterium[1] (i.e., inelastic scattering observed under such circumstances that the neutron is the spectator particle) are just slightly smaller than those predicted by this model. The multiple scattering terms, which are neglected in the simple form of the model discussed here, are expected to account for part of this reduction. The model also predicts that the polarization and triple scattering parameters for elastic nucleon-nucleon scattering and quasielastic scattering are equal. The 145 MeV quasielastic *p-p* and *p-n* polarization measurements of Kuckes and Wilson[1] are in fairly good agreement with earlier elastic data. The quasielastic polarization in *p-p* and *n-p* scattering and the depolarization in *n-p* scattering at 215 MeV[2] and the polarization in *n-p* scattering at 315 MeV[3]

* This work was supported by the National Science Foundation, the U. S. Atomic Energy Commission, and the Higgins Scientific Trust Fund. The experimental work was done in autumn, 1962, while the author was a visitor at Princeton University.

† Present address: The University of Manitoba, Winnipeg, Canada.

[1] A. F. Kuckes, R. Wilson, and P. F. Cooper, Ann. Phys. (N. Y.) **15**, 193 (1961); A. F. Kuckes and R. Wilson, Phys. Rev. **121**, 1226 (1961).

[2] J. H. Tinlot and R. E. Warner, Phys. Rev. **124**, 890 (1961); **125**, 1028 (1962).

[3] O. Chamberlain, E. Segré, R. D. Tripp, C. Wiegand, and T. Ypsilantis, Phys. Rev. **105**, 288 (1957).

[4] L. Cranberg and R. K. Smith, Phys. Rev. **113**, 587 (1959).

[5] C. Wong, J. D. Anderson, C. C. Gardner, J. W. McClure, and M. P. Nakada, Phys. Rev. **116**, 164 (1959); N. A. Vlasov, S. P. Kalinin, B. V. Rybakov, and V. A. Sidorov, Zh. Eksperim. i Teor. Fiz. **38**, 1733 (1960) [translation: Soviet Phys.—JETP **11**, 1251 (1960)].

[6] K. Ilakovac, L. G. Kuo, M. Petravić, and I. Šlaus, Phys. Rev. **124**, 1923 (1961).

[7] S. Kikuchi, J. Sanada, S. Suwa, I. Hayashi, K. Nisimura, and K. Fukunaga, J. Phys. Soc. Japan **15**, 749 (1960); K. Nisimura, *ibid.* **16**, 2097 (1961).

[8] R. M. Frank and J. L. Gammel, Phys. Rev. **93**, 463 (1954).

[9] G. F. Chew and G. C. Wick, Phys. Rev. **85**, 636 (1952).

[10] W. Heckrotte and M. H. MacGregor, Phys. Rev. **111**, 593 (1958); A. B. Migdal, Zh. Eksperim. i Teor. Fiz. **28**, 3 (1955) [translation: Soviet Phys.—JETP **1**, 2 (1955)].

have also been measured. At 215 MeV, the elastic and quasielastic p-p polarizations were found to be equal within 1% accuracy.

The present work was undertaken in order to measure $d^3\sigma/d\Omega_1 d\Omega_2 dE_1$ with protons of much lower bombarding energy than those used in previously reported experiments,[1] and to see if the predictions of either of the above theories could adequately explain the results. In Sec. IV we show that, while neither theory gives an adequate prediction of our experimental data, the disagreement between theory and experiment may be qualitatively explained by physical arguments.

II. EXPERIMENTAL APPARATUS

The Princeton University 35-in. synchrocyclotron[11] was used to produce a 2×10^{-10} A beam of 18.2 MeV protons. The external proton beam was focussed by quadrupole magnets through a $\frac{1}{16}$-in.$\times\frac{1}{4}$-in. defining slit (SD) near the entrance to the 60-in. scattering chamber,[12] and a $\frac{1}{4}$-in.$\times$1-in. antiscattering slit (SA) shielded the particle detectors from the defining slit. The target was a 2.40 mg/cm^2 CD_2 foil.[13] The unscattered beam was collected by a Faraday cup which was connected to a current integrator. Protons scattered by target nuclei through an angle of 7° to the left of the beam entered a double-electrode ion chamber[14] preceded by a suitable absorber. This ion chamber measured the ionization density at two points near the end of the proton range. An error signal obtained from this measurement was used to control the cyclotron magnet current, thereby stabilizing the beam energy to better than ± 0.1 MeV. A drawing of the experimental apparatus appears in Fig. 1.

Protons from the p-d reaction were detected by counters C1 and C2, which consisted of $\frac{1}{8}$-in.-thick NaI(Tl) crystals covered by 0.001-in. Al, coupled to RCA 6655A photomultipliers through Lucite light pipes. The counters were mounted on rotatable arms so that

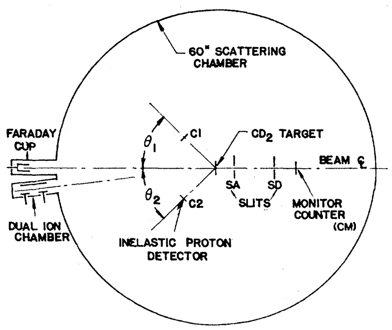

FIG. 1. Top view of experimental apparatus. Note that the incident beam passes under, not through, the monitor counter CM.

[11] A plan view of the cyclotron is given by I. Dayton and G. Schrank, Phys. Rev. **101**, 1358 (1956).
[12] J. L. Yntema and M. G. White, Phys. Rev. **95**, 1226 (1954).
[13] We wish to thank Gordon D. Hiatt of the Eastman Kodak Company, Rochester, New York, for preparing the targets.
[14] G. Schrank, Rev. Sci. Instr. **26**, 677 (1955).

the laboratory scattering angles θ_1 and θ_2 could be changed without opening the scattering chamber. C1 employed a $\frac{3}{4}$-in.$\times 1\frac{1}{2}$-in. rectangular NaI crystal which subtended 0.019 sr at the target. The original scintillator in C2 had the same shape, but its surface deteriorated and all reported measurements were made with a round crystal which subtended 0.013 sr. Both detectors had energy resolutions of approximately 6% (full width at half-maximum) for monoenergetic 10-MeV particles.

The beam was monitored by a plastic scintillation detector (CM) which detected protons elastically scattered through 162° by carbon nuclei in the target. This detector was mounted on top of the tube which contained the beam collimating slits, and a baffle prevented it from detecting protons which back-scattered from the Faraday cup. It was found that the monitor counting rate with the target removed was less than 1% of the rate with the target in place; most of this small residual rate could be attributed to neutron detection. The experiment was performed at a nominal energy of 18.2 MeV so that the monitor counting rate would not change with small variations in the beam energy. The existing data on p-C elastic scattering[15] show that, while at lower energies the cross section at 162° changes rapidly with energy, it is quite flat at 18.2 MeV. This type of monitoring is preferable to beam charge integration since it measures the product of beam intensity and target thickness. Therefore, errors due to nonuniformity of target thickness, wandering of the beam over the target, and incorrect orientation of the target relative to the beam do not appear in the measured cross sections. Nevertheless, the ratio of the monitor and integrator counting rates was found to remain constant within ± 3% over a period of several days.

A block diagram of the electronic circuits appears in Fig. 2. The detectors C1 and C2 were energized by stable negative high-voltage supplies and were connected to preamplifiers just outside the scattering chamber. These were connected to A-8 double-delay-line amplifiers[16,17] through 60-ft, 185-Ω cables. The detection systems were tested for long-term gain stability by observing the pulse height of the elastic p-C peak with a 200-channel pulse-height analyzer. The gain instability did not exceed

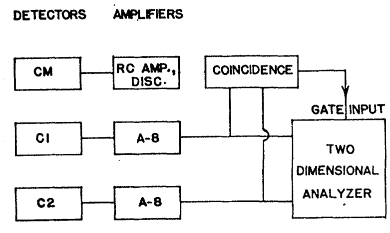

FIG. 2. Electronics circuits.

[15] R. W. Peelle, Phys. Rev. **105**, 1311 (1957); W. Daehnick (private communication).
[16] G. G. Kelly, Inst. Radio Engrs. Natl. Conv. Record **9**, 63 (1957).
[17] We used preamplifier model N-361 and amplifier model N-308, Hamner Electronics Corporation, Princeton, New Jersey.

0.5% in 8 h. Coincidence analysis with resolving time $\tau=30$ nsec was performed by a fast-slow coincidence circuit[18] which sensed the zero-crossover points of the trailing edges of the output pulses from the A-8 amplifiers. Pulses from C1 and C2 were stored in a 32×32 channel two-dimensional pulse-height analyzer[19] which was gated by the coincidence circuit output.

In order to maximize the ratio of true to random-coincidence events, it was necessary to maximize the duty cycle of the synchrocyclotron and minimize the resolving time of the coincidence circuits. During some of the final runs, the auxiliary accelerating electrode[20] was available and the duty cycle could be increased to about 15%. In other runs a 3%–4% duty cycle could be maintained by careful adjustment of the phase of the dequenching oscillator. The rf structure of the cyclotron beam causes particles to arrive in bursts of 10-nsec duration separated by 30-nsec intervals. It follows that, if $\tau=30$ nsec, random coincidences will arise only between particles arriving in the same burst, and no further reduction in random rate can be obtained unless $\tau \ll 10$ nsec. In this experiment, the flight times of detected particles varied by about 5 nsec, and photomultiplier transit time spreads were also expected to be about this large. Therefore, it was necessary to use $\tau=30$ nsec. By detecting elastic p-p coincidences, it was verified that the coincidence circuit operated with full efficiency with $\tau=30$ nsec since, when τ was increased to 200 nsec, the counting rate remained constant within the 1% statistical uncertainty. Since both C1 and C2 detect protons with a large range of energies from p-d breakup, one must verify that the same delay setting will allow simultaneous counter pulses, regardless of their amplitudes, to produce a coincidence output pulse. By driving the two amplifiers with a pulse from a mercury switch pulse generator, attenuated by varying amounts for the two amplifiers, it was found that the total "walk" of the system was less than 10 nsec for the range of pulse heights analyzed in this experiment.

III. EXPERIMENTAL PROCEDURE AND RESULTS

The monitor counter was calibrated by measuring the elastic p-p coincidence rate (n_H) and the monitored elastic p-C scattering rate (n_C) from a 1.3-mg/cm² CH_2 target with $\theta_1=-\theta_2=45°$ (positive scattering angles are to the right of the incident beam, and negative angles are to the left). We thus obtain

$$M=\left(\frac{d\sigma}{d\Omega}\right)_C \Delta\Omega_M=\left(\frac{d\sigma}{d\Omega}\right)_{pp}\frac{\alpha_H n_C \Delta\Omega_1}{\alpha_C n_H}, \qquad (1)$$

where α_H/α_C is the ratio of hydrogen atoms to carbon atoms in the CH_2 target, and $(d\sigma/d\Omega)_{pp}$ is the elastic p-p differential cross section at this energy.[12] For this measurement, C1, the defining counter, was much further from the target than C2. Values of $(d\sigma/d\Omega)_C$ thus obtained were consistent with the known p-C elastic differential cross sections.[15] An energy calibration was next obtained for each detector by accumulating two dimensional energy spectra for elastic p-p scattering with $\theta_1=20°, 25°, \cdots, 70°$; the included angle between counters (θ_{12}) was fixed at 90°. Here the counters were sufficiently far from the target that the kinematic energy broadening was less than the inherent energy resolution of the detectors.

For each analyzed inelastic p-d event the laboratory kinetic energies (E_1,E_2) and coplanar scattering angles (θ_1,θ_2) were determined. At fixed θ_1 and θ_2, E_2 is always either a single valued or a double valued function of E_1. In this experiment, whenever E_2 was double valued, the lower energy (called E_2^-) was below the detection threshold of C1 and C2. Therefore the cross sections reported in this article are $d^3\sigma(\theta_1,\theta_2,E_1,E_2^+)/d\Omega_1 d\Omega_2 dE_1$, where E_2^+ represents either the larger of the two possible values of E_2 (when E_2 is double valued) or the unique value of E_2 (when E_2 is single valued).

The inelastic p-d cross section was measured at several combinations of scattering angles for which one could expect a large yield of events in which both E_1 and E_2 were greater than 3 MeV, and for which intense elastic p-d and p-p groups were not present. (The p-p group comes from the hydrogen impurity in the CD_2 target.) Inelastic p-d events were readily identified in the two-dimensional pulse-height spectra; one such spectrum is shown in Fig. 3. In order to obtain usable counting rates, the detector solid angles were so large that, as a result of kinematic effects, the energy resolution was often poorer than that which would have been obtained if monoenergetic particles had been detected. The number (n_D) of inelastic p-d events in the interval between $E_1-\frac{1}{2}\Delta E_1$ and $E_1+\frac{1}{2}\Delta E_1$ (where ΔE_1 denotes the width of one channel in the E_1 direction) was obtained by adding the events from a sufficiently large number of channels in the E_2 direction that the error resulting from omitting events was much less than the statistical error. The cross section was then computed from the equation

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1}=\frac{\alpha_C n_D M}{\alpha_D n_C \Delta\Omega_1 \Delta\Omega_2 \Delta E_1}, \qquad (2)$$

where α_C/α_D is the ratio of carbon to deuterium atoms in the CD_2 target.

Considerations of target composition, detector efficiency, and detector energy threshold indicated that real background counting rates due to competing nuclear reactions [primarily $C^{12}(p,p)3\alpha$, $C^{12}(p,2p)B^{11}$, and $C^{13}(p,pn)C^{12}$] should be negligible. Runs with a CH_2 target confirmed this conclusion. Random coincidence

[18] Model 801, Cosmic Radiation Laboratories, Bellport, New York.

[19] Model 150 with typewriter printout, Nuclear Data, Inc., Madison, Wisconsin.

[20] H. O. Funsten, A. Lieber, R. Roberson, and R. Sherr, Bull. Am. Phys. Soc. 8, 13 (1963).

rates were nearly always less than 10% of the total counting rates for $\theta_1=-\theta_2$, but were sometimes higher for other geometries for which the cross sections were lower. Since successive rf bursts may have different particle intensities, the random rates were computed from the same two-dimensional pulse-height spectra as the total rates, rather than from spectra recorded with the signal from one counter delayed by an integral number of rf periods. The random coincidence rate (R_{ij}) for a channel ij (in which $E_1=E_i$, $E_2=E_j$) which contained both real and random-coincidence events was computed from the equation[21]

$$R_{ij}=(R_{im}R_{jn})/R_{mn}\,, \tag{3}$$

where the channels im, jn, and mn were chosen so that they contained only random events. A test of detector gain stability was obtained through pulse-height analysis of the group of random coincidence events in which C1 detected an inelastic proton from $C^{12}(p,p')C^{12*}$ (4.4 MeV state). This test showed that the gain remained constant to better than 2% throughout the experiment.

The data are presented in Figs. 4 and 5 (data tabulations will be supplied upon request). The experimental errors include only statistical uncertainties (including those arising from the subtraction of random coincidence events). A total systematic error estimated at 5% arises from uncertainties in target composition, detector solid angles, the elastic p-p cross section, and from the small gain instability of the monitor. In addition, we have no precise measurements of the differential linearity of the two-dimensional analyzer for particle pulses. If this were worse than our other experimental errors, it would influence the accuracy of the differential cross section at a given energy E_1, but not the integrated value of the cross section over a large interval (say 4 or 5 MeV) in E_1.

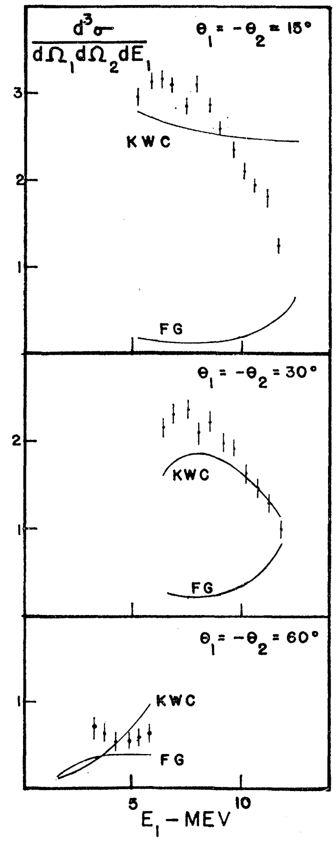

FIG. 4. Experimental cross sections in mb/sr²–MeV and theoretical predictions for $\theta_1=-\theta_2=15°$, 30° and 60°. Curves marked FG were calculated from the theory of Frank and Gammel (Ref. 8). Those marked KWC were calculated from the theory of Kuckes, Wilson, and Cooper (Ref. 1); the 30° KWC prediction has been divided by 4.

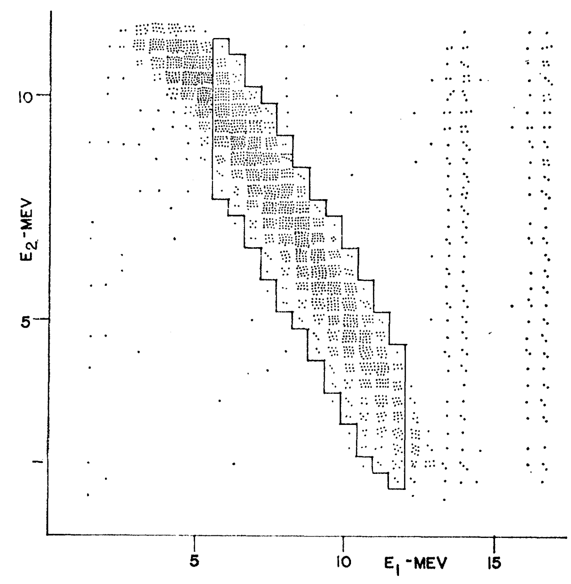

FIG. 3. Two-dimensional pulse-height spectra at $\theta_1=-\theta_2=20°$. Each dot represents five events. Analyzed events attributed to inelastic p-d scattering lie inside the enclosed area. The column of dots near $E_1=14$ MeV results from accidental coincidence events in which C1 detects an inelastically scattered proton from $C^{12}(p,p')C^{12*}$ (4.4 MeV). Those near 17 MeV are caused by protons from elastic p-d scattering.

TABLE I. Experimental values of $d^2\sigma/d\Omega_1 d\Omega_2$ at 18.2 MeV (our data) and 20.2 MeV (Ref. 22). $d^2\sigma/d\Omega_1 d\Omega_2$ is defined in Eq. (4), and is expressed in units of mb/sr². The 18.2-MeV errors include statistical uncertainty and an estimated 5% systematic error.

θ_1	θ_2	$d^2\sigma$ (18.2 MeV)	$d^2\sigma$ (20.2 MeV)
20°	−20°	8.8±0.5	12.3±1.3
30°	−30°	7.5±0.4	13.6±1.0
40°	−40°	5.7±0.3	13.5±1.1

In Table I these data are compared with those of Doub,[22] who measured the quantity

$$\frac{d^2\sigma}{d\Omega_1 d\Omega_2}=\int_{E_l}^{E_2}\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1}dE_1\,, \tag{4}$$

[21] R. W. Detenbeck, Ph.D. thesis, Princeton University, 1962 (unpublished).

[22] W. B. Doub, Ph.D. thesis, University of California, Los Angeles, 1956 (unpublished). We wish to thank Dr. Doub for permission to quote these unpublished data and Professor B. T. Wright for bringing them to our attention.

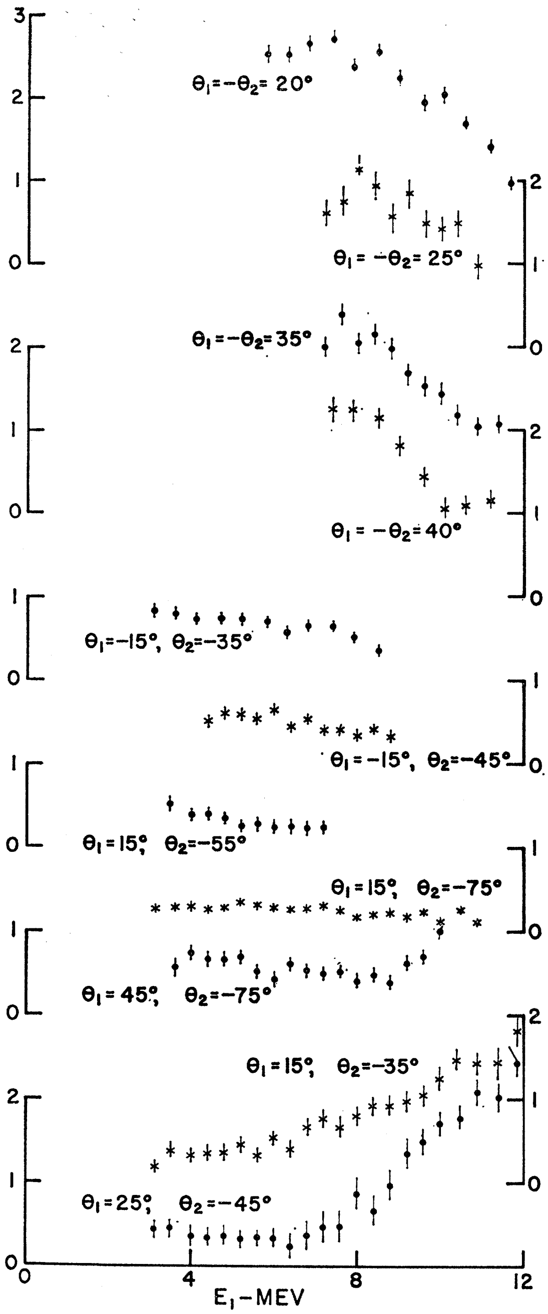

FIG. 5. Experimental cross sections in mb/sr²–MeV for scattering angles at which theoretical calculations were not made.

with 20.2-MeV incident protons. (E_t is defined such that $E_2=2.76$ MeV when $E_1=E_t$.) The differential cross sections at 20.2 MeV would be expected to be somewhat larger than those at 18.2 MeV since the total cross section is expected to increase with increasing energy and, in addition, the energy range defined in Eq. (4) includes a larger fraction of the total available phase space at 20.2 than at 18.2 MeV. However, the magnitude of the difference is surprising, as is the fact that the 18.2 MeV cross section decreases by 35% in going from $\theta_1=-\theta_2=20°$ to 40° while the 20.2 MeV cross section remains constant. Also, Kikuchi *et al.*[7] found that $d^2\sigma/d\Omega dE$ for single protons from p-d breakup by 14 MeV incident protons decreased with increasing scattering angle. It would therefore be very useful to have more experimental data at nearby energies as well as a successful theoretical treatment which gave the energy dependence of the cross section.

IV. THEORETICAL DISCUSSION

In the Frank-Gammel[8] theoretical treatment the following approximations are made: The impulse approximation is used; the nuclear forces are taken to have zero range; the ground-state deuteron wave function is assumed to have its asymptotic form everywhere; and the excited (continuum) deuteron is assumed to be in an s state after breakup. Their equation (36b) then gives the inelastic cross section in the c.m. system. By using the conservation laws to transform the k's in this equation to lab energies and momenta, and integrating over the magnitude of the momentum of the second proton, one obtains the predicted inelastic cross section.[23]

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1}=\frac{9E_2}{8\pi^2}\left(\frac{E_1}{2E_0E_b}\right)^{1/2}\left\{\left(\frac{1}{D_{t1}}+\frac{0.1587}{D_{s1}}\right)\left(\frac{d\sigma}{d\Omega}\right)_1^{\mathrm{el}}+\left(\frac{1}{D_{t2}}+\frac{0.1587}{D_{s2}}\right)\left(\frac{d\sigma}{d\Omega}\right)_2^{\mathrm{el}}\right\}$$
$$\times[E_1(\cos^2\theta_{12}-4)+E_0\cos^2\theta_2-2E_b+2(E_0E_1)^{1/2}(2\cos\theta_1-\cos\theta_2\cos\theta_{12})]^{-1/2},\quad(5)$$

where

$$D_{t1}=E''\csc^2\delta_t=(\tfrac{2}{3}E_0-E_b-\tfrac{3}{2}E_1')\csc^2\delta_t,\quad(6)$$

and D_{s1}, D_{t2}, and D_{s2} are similarly defined. δ_s and δ_t are the singlet and triplet n-p s-wave phase shifts at energy E'', which are calculated from effective range theory. E_0, E_b, and E_1' denote the lab kinetic energy of the incident proton, the deuteron binding energy, and the c.m. kinetic energy of the proton detected by C1. The first term inside curly brackets gives the contribution from events in which C1 detects the "scattered" proton and C2 detects the proton "ejected" from the deuteron, and the second term represents the opposite case. We take $(d\sigma/d\Omega)_1^{\mathrm{el}}$ to be the experimentally measured c.m. cross section for elastic p-d scattering[24] at such an angle that the lab momentum of an elastically scattered proton would be equal to that of the inelastically scattered proton detected by C1. This is done since the matrix elements for both elastic and inelastic scattering contain integrals [J_2 in Eqs. (27) and (28), Ref. 8] which, apart

[23] This transformation was derived in collaboration with Dr. J. L. Gammel.

[24] D. O. Caldwell and J. R. Richardson, Phys. Rev. 98, 28 (1955).

from constant factors, depend only upon the magnitude of the lab momentum vector of the scattered proton.

Fig. 4 contains three sets of experimental data and the corresponding theoretical predictions. The calculated Frank-Gammel cross sections are seen to be nearly always too low.[25] At $\theta_1=-\theta_2=15°$ and at $\theta_1=-\theta_2=30°$ the two protons share nearly all of the available lab kinetic energy, and on physical grounds one would expect most scattering events to result from a large momentum transfer from the incident proton to the proton in the deuteron. However, the contribution of the p-p potential vanishes in this treatment, and so it is not surprising that we predict too small a cross section. At $\theta_1=-\theta_2=60°$ the neutron receives a larger kinetic energy, and here the theory does predict a somewhat larger fraction of the average measured cross section.

The spectator model assumptions of Kuckes, Wilson, and Cooper[1] are these: The impulse approximation is made; the interaction of the incoming proton with one nucleon in the deuteron (in our case, the neutron) is ignored; the deuteron ground state is described by a Hulthén wave function; and all unbound nucleons have plane wave functions. They obtain the equation[26]

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1}=\frac{4}{\pi^2}\left(\frac{2E_bE_cE_1}{E_0}\right)^{1/2}\frac{(E_b^{1/2}+E_c^{1/2})^3E_2(d\sigma/d\Omega)_{pp}}{(E_b+2E_3)^2(E_c+2E_3)^2}$$
$$\times[2E_2^{1/2}-E_0^{1/2}\cos\theta_2+E_1^{1/2}\cos\theta_{12}]^{-1}, \quad (7)$$

where $E_c=59.8$ MeV and E_3 is the lab energy of the neutron after breakup. $(d\sigma/d\Omega)_{pp}$ is the measured c.m. p-p differential cross section[12] at the angle which gives the same momentum transfer as the corresponding inelastic event. The results of our calculations appear in Fig. 4. At 30° the calculated cross section has about the same energy dependence as the data, but is too large by a factor of almost 4 in absolute magnitude. At 40° (not shown in Fig. 4), the predicted cross sections are 8 times too large. This discrepancy may, in part, arise from the use of the impulse approximation.

The impulse approximation asserts that the nucleons in the deuteron are so far apart that the incident nucleon seldom interacts strongly with both at once, and that the amplitude of the incident wave at the location of each nucleon is nearly the same as if that nucleon were alone. According to the criteria of Chew and Wick,[9] these requirements should be at least qualitatively satisfied at this energy. The third requirement is that the binding potential be negligible during the decisive phase of the collision (equivalently, that the collision time τ_c be short in comparison with the deuteron period). The quantitative statement is $U\tau_c \ll \hbar$, where U is the potential energy of a nucleon in the deuteron; at this energy $U\tau_c \approx 2\hbar$, so the assumption fails badly. Thus, while the spectator model does take into account the p-p interaction, which is expected to produce most of the scattering events observed in our geometry, it overestimates the scattering cross section by neglecting the effect of the deuteron binding.

A more rigorous treatment of nucleon-deuteron inelastic scattering is evidently needed since neither the zero-range approximation nor the spectator model can adequately account for the data presented here. A more successful theory would probably have to consider the effects of all the nucleon-nucleon potentials and would, in addition, have to take multiple scattering into consideration. Furthermore, the plane wave assumption has doubtful validity at the bombarding energy used in this experiment; better agreement might well be obtained by using more realistic wave functions. The precise shape of the nucleon-nucleon potentials is probably not too critical for this purpose, since both the incident and the scattered nucleons have $\lambda\!\!\!^{-}$ approximately equal to or slightly greater than the range of nuclear forces.

ACKNOWLEDGMENTS

I wish to thank Professor Rubby Sherr for the hospitality extended during my stay at Princeton, and for his advice and encouragement throughout the experiment. I am indebted to Dr. John Gammel for advice and help with the theoretical part of the work. A. Emann, S. Silbey, and other members of the Princeton cyclotron group provided technical assistance, and Hugh Lauer assisted in constructing equipment and collecting data. Computations and data analysis were done by Margaret Dunlap, Steven Eckroad, Lawrence Rippere, and Charles Smith.

---

[25] The calculated cross sections would be further reduced by the correction for Coulomb penetration (Ref. 8, p. 469); however, since this correction would never exceed 15%, it has not been made.

[26] Here Eq. (9) of Ref. 1 has been corrected by multiplying the right side by $(E_0E_1)^{-\frac{1}{2}}$.